Online Appendices (Appendix 3 from Main Paper)

for

The Strategic Implications of Scale in Choice-Based Conjoint Analysis

by

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1. Numerical example for profits, market shares, equilibrium prices, first-order conditions and second-order conditions for stylized model.
2. Numerical example of market-research decisions by a sophisticated follower.
3. Attribute descriptions and example choice tasks for dormitory CBC study.
4. Brief summary of the McFadden-based (stylized-model based), Sonnier, Ainslie, and Otter (2007), and Allenby, et al. (2014) HB CBC specifications.
5. Comparison of estimates for scale adjustment factors from the McFadden-based (stylized-model based), Sonnier, Ainslie, and Otter (2007), and Allenby et al. (2014) specifications. Including alternative estimations accounting for gender, for split-sample, for split choice task, for a mixtures of normal distributions.
6. Posterior distributions for scale adjustment factors and attribute importances.
7. Posterior WTP estimates for McFadden-based (stylized-model based), Sonnier, Ainslie, And Otter (2007) and Allenby et al. (2014) specifications.
8. Additional citations: Five *Marketing Science* papers that discuss scale explicitly

Online Appendix 3.1.
Numerical Example for Profits, Market Shares, Equilibrium Prices, First-Order Conditions and Second-Order Conditions for Stylized Model.

We illustrate the formal insights with a numerical example in which $\beta^h = 2$, $\beta^\ell = 1$, $u_o = 1$, and $R = 0.55$. We obtain the fixed-point price equilibria by simple iteration combined with grid search. In all cases, we check that the second-order and cross-partial conditions are satisfied. (R program available from the authors.)

3.1.1. Representative Equilibria for Various Levels of Scale

Table 3.1.1 reports price equilibria for differentiated strategies (rs) for different values of scale (γ^{true}). In practice we expect γ^{true} to be the order of magnitude of the partworths, but to illustrate key issues we vary γ^{true}over a wider range. For very small values of true scale ($\gamma^{true} = 0.05$), the market is less sensitive to price allowing firms to price highly and earn substantial profits. Profits and prices decrease with γ^{true}. For very large values of γ^{true}, the innovator's share in segment S (P^*_{S1rs}) approaches zero as does the follower's share in Segment R (P^*_{R2rs}). The market becomes more segmented when true scale increases.

Table 3.1.2 reports equilibria for undifferentiated strategies (rr) using the same values of true scale as in Table 3.1.1. The second-order conditions are always satisfied for undifferentiated strategies. Low true scale implies high prices and profits. When true scale is large, the market is very sensitive to price and the shares in Segment R approach 50%. If the firms do not differentiate, the high price sensitivity due to large scale drives profits to zero. The last two columns of Table 3.1.2 compare profits between a differentiated (rs) strategy and an undifferentiated strategy (rr). For low scale (below 1.0), strategy rr is more profitable than rs for the follower. This is shown in a **red bold font**.

Table 3.1.3 reports the shares of the outside option in Segment R, Segment S, and overall for both a differentiated market and an undifferentiated market.

Table 3.1.1. Prices, Shares, Profits, and Second-order Conditions: Differentiated Market

Scale	Prices		Shares in Segment R		Shares in Segment S		Profits		Second Order Conditions	
γ^{true}	p^*_{1rs}	p^*_{2rs}	P^*_{R1rs}	P^*_{R2rs}	P^*_{S1rs}	P^*_{S2rs}	π^*_{1rs}	π^*_{2rs}	$\frac{\partial^2 \pi^*_{1rs}}{\partial p^2_{1rs}}$	$\frac{\partial^2 \pi^*_{2rs}}{\partial p^2_{2rs}}$
0.05	24.625	24.603	0.192	0.183	0.183	0.192	4.622	4.600	-0.009	-0.009
0.50	2.588	2.564	0.261	0.160	0.158	0.264	0.556	0.531	-0.103	-0.107
0.60	2.190	2.166	0.278	0.155	0.146	0.299	0.485	0.459	-0.132	-0.126
0.70	1.909	1.885	0.295	0.149	0.139	0.316	0.435	0.408	-0.157	-0.149
0.80	1.701	1.677	0.311	0.143	0.133	0.334	0.398	0.370	-0.184	-0.173
0.90	1.543	1.519	0.328	0.136	0.195	0.275	0.371	0.342	-0.212	-0.198
1.0	1.418	1.394	0.345	0.130	0.126	0.352	0.349	0.316	-0.137	-0.144
2.0	0.923	0.905	0.501	0.070	0.067	0.511	0.282	0.243	-0.491	-0.573
3.0	0.817	0.807	0.614	0.031	0.030	0.621	0.287	0.240	-0.808	-0.589
4.0	0.787	0.783	0.692	0.013	0.013	0.696	0.304	0.251	-1.200	-1.481
5.0	0.779	0.778	0.747	0.005	0.005	0.747	0.322	0.264	-1.639	-2.018
10	0.805	0.805	0.876	0.000	0.000	0.876	0.388	0.317	-3.938	-4.815
20	0.861	0.861	0.942	0.000	0.000	0.942	0.446	0.365	-8.477	-10.36
200	0.974	0.974	0.995	0.000	0.000	0.995	0.533	0.436	-89.54	-109.4

Table 3.1.2. Prices, Shares, Profits, and Relative Profits: Undifferentiated Market

Scale	Prices		Shares in Segment R		Shares in Segment S		Profits		Relative Profits	
γ^{true}	p^*_{1rr}	p^*_{2rr}	P^*_{R1rr}	P^*_{R2rr}	P^*_{S1rr}	P^*_{S2rr}	π^*_{1rr}	π^*_{2rr}	$\pi^*_{1rs} - \pi^*_{1rr}$	$\pi^*_{2rs} - \pi^*_{2rr}$
0.05	24.619	24.619	0.190	0.190	0.184	0.184	4.618	4.618	0.004	**-0.018**
0.50	2.553	2.553	0.240	0.240	0.179	0.179	0.542	0.542	0.014	**-0.011**
0.60	2.147	2.147	0.251	0.251	0.178	0.178	0.468	0.468	0.017	**-0.009**
0.70	1.858	1.858	0.262	0.262	0.176	0.176	0.415	0.415	0.020	**-0.007**
0.80	1.642	1.642	0.272	0.272	0.175	0.175	0.375	0.375	0.023	**-0.005**
0.90	1.474	1.474	0.283	0.283	0.173	0.173	0.345	0.345	0.026	**-0.002**
1.0	1.341	1.341	0.294	0.294	0.172	0.172	0.320	0.320	0.029	0.004
2.0	0.744	0.744	0.385	0.385	0.156	0.156	0.209	0.209	0.072	0.034
3.0	0.539	0.539	0.444	0.444	0.142	0.142	0.166	0.166	0.121	0.074
4.0	0.425	0.425	0.476	0.476	0.134	0.134	0.137	0.137	0.167	0.114
5.0	0.349	0.349	0.491	0.491	0.130	0.130	0.114	0.114	0.208	0.150
10	0.177	0.177	0.500	0.500	0.127	0.127	0.059	0.059	0.329	0.258
20	0.089	0.089	0.500	0.500	0.127	0.127	0.029	0.029	0.416	0.335
200	0.009	0.009	0.500	0.500	0.127	0.127	0.003	0.003	0.530	0.433

Table 3.1.3. Shares of the Outside Option

Scale	Outside Option Segment R		Outside Option Segment S		Outside Option Net Share	
γ^{true}	P^*_{ROrs}	P^*_{ROrr}	P^*_{SOrs}	P^*_{SOrr}	P^*_{Ors}	P^*_{Orr}
0.05	0.625	0.626	0.625	0.626	0.625	0.626
0.50	0.581	0.581	0.576	0.581	0.579	0.581
0.60	0.576	0.571	0.546	0.571	0.563	0.571
0.70	0.566	0.562	0.535	0.562	0.552	0.562
0.80	0.556	0.553	0.523	0.553	0.541	0.553
0.90	0.477	0.544	0.589	0.544	0.527	0.544
1.0	0.529	0.534	0.518	0.534	0.524	0.534
2.0	0.432	0.459	0.419	0.459	0.426	0.459
3.0	0.356	0.414	0.348	0.414	0.352	0.414
4.0	0.295	0.390	0.291	0.390	0.293	0.390
5.0	0.248	0.379	0.248	0.379	0.248	0.379
10	0.124	0.373	0.124	0.373	0.124	0.373
20	0.058	0.373	0.058	0.373	0.058	0.373
200	0.005	0.373	0.005	0.373	0.005	0.373

Online Appendix 3.2.
Numerical Example of Market-Research Decisions by a Sophisticated Follower

Decisions on CBC-based market research spending depend upon Equations 6 and 7. Suppose, for the sake of illustration, that the market potential is 10 million units and that prices are scaled in dollars. Suppose further that the follower anticipates that the higher-quality CBC study reveals the true scale, $\gamma^{higher} = \gamma^{true}$. It will act on the γ^{true} that is revealed. It uses its prior to anticipate the γ^{true} that will be revealed. The lower-quality CBC study does not reveal γ^{true}, therefore the follower must act based on its prior. If the follower chooses the lower-quality CBC study, the follower bases its positioning strategy based on expected profits, integrating over $g(\gamma^{true})$. The calculations are given in Table 3.2.1.

Based on Table 3.2.1, an undifferentiated strategy has a higher expected value than a differentiated strategy, hence the follower using a lower-quality CBC study would choose r as per Equation 6. If the follower invests in the higher-quality CBC study, the follower can choose its strategy (r or s) depending upon the γ^{true} it observes. The follower's decision after observing $\gamma^{true}$is indicated by the "Best Strategy" column. Choosing the best strategy for each realized γ^{true} yields higher expected profits ($5,034,722) compared to the best strategy based only on the lower-quality study ($4,981,407). The difference, $53,315, is the most that a sophisticated follower would pay for a higher-quality CBC study.

Table 3.2.1 also illustrates that a naïve follower can make strategic errors. Suppose the follower invests in a lower-quality CBC study that tells the firm (incorrectly) that $\gamma^{true} = 0.1$. Believing and acting on the lower-quality CBC study, the follower would choose not to differentiate (r) and forecast a profit of over $23.5M. If true scale were really $\gamma^{true} = 2.0$, then the firm would (1) position the product incorrectly (r rather than s), (2) bear an opportunity cost of $335,010, and (3) not realize anywhere near its anticipated profit ($2.1M vs. $23.5M).

Table 3.2.1. Illustration of the Follower's Decisions and Outcomes Based on Either a Lower-Quality CBC Study (Columns 3&4) or a Higher-Quality CBC Study (Column 6)

Prior, $g(\gamma^{true})$	True Scale, γ^{true}	Follower Chooses s Based on Lower-Quality CBC Study	Follower Chooses r Based on Lower-Quality CBC Study	Best Strategy After γ^{true} Revealed	Follower Chooses r or s after Higher-Quality CBC Study
0.03	0.1	$23,337,834	$23,509,998	***r***	$23,509,998
0.03	0.2	$12,027,032	$12,186,344	***r***	$12,186,344
0.08	0.3	$8,275,610	$8,420,431	***r***	$8,420,431
0.08	0.4	$6,414,787	$6,543,437	***r***	$6,543,437
0.08	0.5	$5,310,777	$5,421,558	***r***	$5,421,558
0.08	0.6	$4,585,625	$4,676,841	***r***	$4,676,841
0.08	0.7	$4,077,318	$4,147,275	***r***	$4,147,275
0.08	0.8	$3,704,817	$3,751,862	***r***	$3,751,862
0.08	0.9	$3,423,066	$3,445,561	***r***	$3,445,561
0.08	1.0	$3,204,993	$3,201,369	***s***	$3,204,993
0.03	1.1	$3,033,356	$3,002,089	***s***	$3,033,356
0.03	1.2	$2,896,596	$2,836,255	***s***	$2,896,596
0.03	1.3	$2,786,715	$2,695,922	***s***	$2,786,715
0.03	1.4	$2,697,959	$2,575,425	***s***	$2,697,959
0.03	1.5	$2,626,085	$2,470,611	***s***	$2,626,085
0.03	1.6	$2,567,891	$2,378,368	***s***	$2,567,891
0.03	1.7	$2,520,907	$2,296,323	***s***	$2,520,907
0.03	1.8	$2,483,216	$2,222,635	***s***	$2,483,216
0.03	1.9	$2,453,264	$2,155,856	***s***	$2,453,264
0.03	2.0	$2,429,844	$2,094,834	***s***	$2,429,844
Expected Profits		$4,975,580	$4,981,407		$5,034,722

Online Appendix 3.3

Attribute Descriptions and Example Choice Tasks For Dormitory CBC Study.

CBCfeatures1 EDIT

The dormitory living arrangements that will be shown to you may vary in the features within each of the following categories:

1. Unit type
2. Commute time
3. Access to grocery stores and bars/cafes/restaurants
4. Bedroom size
5. Building amenities
6. Parking
7. Monthly rent per household

The following screens will provide greater detail about the features of dormitory living arrangements. We will delay the forward arrow slightly to enable you to read the screen.

1. Unit type

The family dormitory living options come in three types of units. The pictures are illustrative; the units at your college or university may differ in details.

Studio 	Private bathroom, compact kitchen There are no walls separating sleeping and living areas.
1-bedroom apartment 	Private bathroom, full-size kitchen. The unit includes a living room.
2 bedroom apartment 	Private bathroom, full-size kitchen The unit includes a living room.

Please assume that:

- If the unit comes with a living room, the space is large enough to fit a couch and TV.
- If the unit comes with a kitchen, large user-provided appliances are not allowed.
- If the unit comes with a full-size kitchen, the kitchen includes a full-size refrigerator, regular electric stove and oven, and sink.

2. Commute time

The dormitories vary on their distance from your classrooms, laboratories, office, and other academic buildings. The three options are:

10 min walk, 3-5 min bike. 	About 0.5 or fewer miles away.
20 min walk, 6-10 min bike. 	About 1 mile away.
20 min bike, driving may be necessary. 	About 3 mile away.

Please assume that:

- Commute time represents the time it takes you to get to your classroom, laboratory, office, or other academic building.
- Times are based on average walking, biking, or driving speeds.

3. Access to grocery stores and bars/cafes/restaurants

There are three options describing access from your dormitory living arrangement. The pictures are illustrative, the grocery stores, bars, or restaurants at your college or university may differ in details.

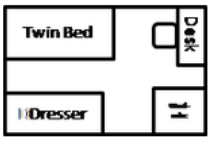
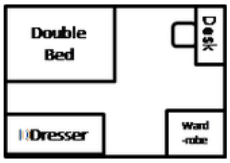
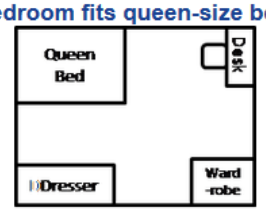
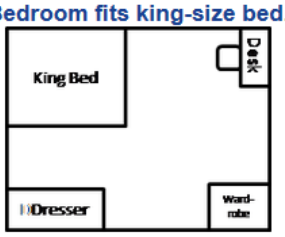
None 	There is no grocery store or bars/cafes /restaurants in the neighborhood.
Grocery store nearby 	There is a grocery store in the neighborhood, but no bars/cafes/restaurants nearby.
Grocery store & bars/cafes /restaurants nearby 	There is a grocery store as well as bars/cafes /restaurants in the neighborhood.

Please assume that:

- The grocery store is an affordable grocery market.
- "Nearby" means the location is a short walk.

4. Bedroom size

There are four options describing the size of the bedrooms in your dormitory living arrangement. The pictures are illustrative, the units at your college or university may differ in details.

Bedroom fits twin-size bed. 	The bedroom is about 150 sq. ft. (14 sq. m.).
Bedroom fits double-size bed. 	The bedroom is about 200 sq. ft. (18.5 sq. m.).
Bedroom fits queen-size bed. 	The bedroom is about 250 sq. ft. (23 sq. m.).
Bedroom fits king-size bed. 	The bedroom is about 300 sq. ft. (28 sq. m.).

Please assume that:

- All of the bedrooms accommodate a desk, chair, dresser, basic closet, and room to walk around.
- All bedrooms can come furnished or unfurnished at your discretion at the same cost.
- For a studio, this size represents about two-thirds of the total space. The remaining space is made up of the kitchen and bath.

5. Building amenities

There are three options describing building amenities in your dormitory living arrangement: none, some, and many. The pictures are illustrative, the amenities at your college or university may differ in details.

None 	No amenities
Some 	Small community lounge, small fitness center, outdoor area, front desk, same-day maintenance service,
Many 	Large community lounge, large fitness center, study lounge, music room, recreation/game room, outdoor area, barbecue in outdoor area, front desk, same-day maintenance service

Please assume that:

- All amenities are free to use for residents.
- All amenities can only be used by residents and their guests.
- If available, the fitness center will be in a central location in the residence
- All dormitories will include a laundry room in the basement of the building with sufficient washers and dryers.
- Bike storage will be available with all buildings.

ParkingRent EDIT

6. Parking

There are three options describing parking in your dormitory living arrangement. The pictures are illustrative, the facilities at your college or university may differ in details.

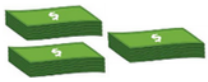
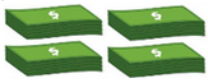
No parking 	There is no parking nearby
Paid uncovered parking 	There is uncovered parking nearby for $160/month.
Paid covered parking 	There is covered parking nearby for $190/month. (Some facilities have a covered garage.)

Please assume that:

- If parking is offered, there is sufficient parking to accommodate all residents who request a spot.
- "Nearby" means the location is within a short walk.

7. Monthly rent per household

There are five options describing monthly rent at your dormitory living arrangement.

$500 
$1000 
$1500 
$2000 
$2500 

Please assume that:

- The price listed is per household.
- The price listed includes utilities and Wi-Fi.
- There are no security or cleaning deposits or last month rent required upfront.

Imagine that you are in the market and making the decision of where to live as at your college or university. You are choosing where would be best for you based on a number of alternatives.

If there were no other options available AND all other features not mentioned in the exercise were the same across the dormitory living arrangements shown, which would you choose?

You may click on any attribute to review the levels at any time.

Choose by clicking one of the buttons below:

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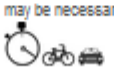
Unit Type	Studio, private bathroom, compact kitchen 	2-bedroom apartment, private bedroom, full-size kitchen 	1-bedroom apartment, private bedroom, full-size kitchen 	Studio, private bathroom, compact kitchen 
Commute Time	20 min walk, 6-10 min bike 	20 min bike, driving may be necessary 	10 min walk, 3-5 min bike 	10 min walk, 3-5 min bike 
Access to Grocery Stores and Bars/Cafes /Restaurants	Nothing in Neighborhood 	Grocery store nearby 	Grocery store & bars/cafes /restaurants nearby 	Nothing in Neighborhood 
Bedroom Size	Bedroom fits twin-size bed 	Bedroom fits queen-size bed 	Bedroom fits king-size bed 	Bedroom fits double-size bed 
Building Amenities	Some 	Many 	None 	None 
Parking	Paid uncovered parking 	No parking 	Paid covered parking 	No parking 
Rent per Household	$1000 	$2000 	$2500 	$500 
	Select	Select	Select	Select

Given your knowledge of the real-estate market, would you actually be willing to live in the dormitory living arrangement you chose at the price indicated?

(Please assume that if you choose not to live at the option you chose above, you would be choosing to live off-campus at a residence you found on your own.)

- ◯ Yes, I would be live in the dormitory living arrangement that I chose above.
- ◯ No, I would rather live off-campus than live in the dormitory living arrangement I chose above.

Online Appendix 3.4.
Brief Summary of the McFadden-Based (Stylized-Model Based), Sonnier, Ainslie, And Otter (2007), and Allenby, et al. (2014) HB CBC Specifications.

Stylized model (McFadden 2014 based) specification. In the stylized model specification we model consumer i's utility u_{ij} for product profile j as:

$$u_{ij} = \sum_{k=1}^{K} \gamma_i(\beta_{ki} a_{jk} - p_j) + \epsilon_{ij}$$

where a_{jk} refers to the level of attribute k (effect-coded) and p_j to the price (in \$150 units). We apply a multinomial logit model to consumer i's choices, y_i, given attribute levels, price, and preference parameters, within a hierarchical Bayes framework. The β_i's and $\ln(\gamma_i)$ are assumed multivariate normally distributed with mean θ and covariance matrix V. The second-stage prior is the standard Normal-Inverted-Wishart conditionally conjugate prior. The hierarchical model is then specified as:

$$y_i | a_j, p_j, \beta_i, \gamma_i$$

$$\beta_i, \ln(\gamma_i) \sim N(\theta, V)$$

$$V^{-1} \sim W(\nu, \nu V_0)$$

We apply Allenby et al.'s "default" settings (p. 438) and use a relatively diffuse prior with the following parameters: $\theta = 0$, $\nu = \dim(\beta_i) + 6$, and $V_0 = I$. Consistent with Allenby et al. (2014) we lower the diagonal element of V_0 corresponding to γ_i to 0.5 to account for its logarithmic scale.

Sonnier, Ainslie, and Otter (2007) specification. Sonnier et al. model consumer i's utility u_{ij} as:

$$u_{ij} = \sum_{k=1}^{K} \frac{1}{\mu_i} (\beta_{ki} a_{jk} - p_j) + \epsilon_{ij}$$

In the first stage prior the β_{ki}'s are assumed normally distributed, $\ln(\mu_i)$ is assumed normally distributed. The second-stage prior and hyper-priors remain consistent to the stylized model (replacing the hyper-prior for γ_i with an equivalent hyper-prior for μ_i).

Allenby, Brazell, Howell, and Rossi (2014) specification. Allenby et al. do not employ a scaling

parameter but estimate a price parameter, β_{pi}, in their utility specification:

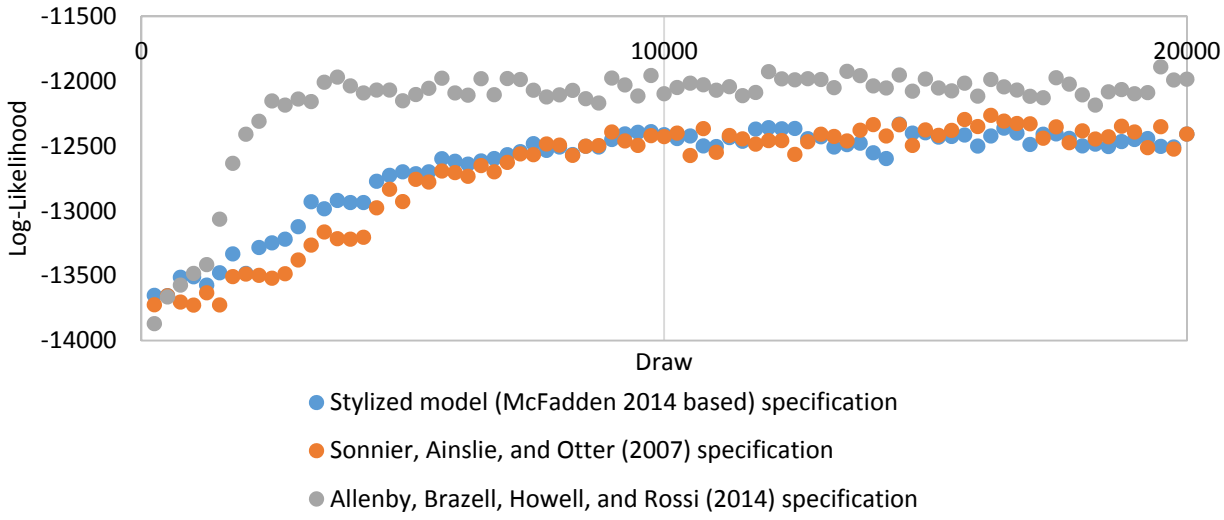
$$u_{ij} = \sum_{k=1}^{K} \beta_{ki} a_{jk} - \beta_{pi} p_j + \epsilon_{ij}$$

The β_{ki}'s are assumed normally distributed, $\ln(\beta_{pi})$ is assumed normally distributed. The second-stage prior and hyper-priors remain consistent to the stylized model (replacing the hyper-prior for γ_i with an equivalent hyper-prior for β_{pi}).

We tested other prior specifications, e.g., $\nu = \dim(\beta_i) + 16$, and the results remained consistent (see Appendix 3.5.).

HB settings. All settings not specified by Allenby et al. followed standard procedures, e.g., as in Sawtooth Software (2015). For example, we used 10,000 burn-in iterations and a subsequent 10,000 iterations to draw partworths, from which we kept every 10th draw. The iteration time series show that the process converged after the burn-in phase for all specifications (see Figure 3.4.1 for log-likelihood statistics). See Appendix_4_Supplement_Iteration_Statistics.xlsx. All summaries, profits, and other reported quantities are based on the posterior distributions.

Figure 3.4.1. Iteration time series of the three CBC HB Specifications



Online Appendix 3.5.
Comparison of Estimates for Scale Adjustment Factors from the McFadden-Based (Stylized-Model Based), Sonnier, Ainslie, And Otter (2007), and Allenby et al. (2014) Specifications. Including Alternative Estimations Accounting for Gender, for Split-Sample, for Split Choice Task, for a Mixtures of Normal Distributions.

The detailed estimates are contained in a companion spreadsheet,

Appendix_5_Comparison_of_Posterior_Scale_Adjustment Estimates.xlsx.

Online Appendix 3.6.
Posterior Distributions for Scale Adjustment Factors and Attribute Importances

The full posterior distributions and summaries for the scale adjustment factors, attribute importances, and individual posterior means are contained in a companion spreadsheet,

Appendix_6_Table_3_Relative_Importances.xlsx and Appendix_6_Table_4_Scale_Adjustments.xlsx. See also Appendix_6_Individual_Posterior_Means_of_Random_Parameters.xlsx.

Online Appendix 3.7.
Posterior WTP Estimates for McFadden-Based (Stylized-Model Based), Sonnier, Ainslie, And Otter (2007) and Allenby et al. (2014) Specifications.

The detailed estimates are contained in a companion spreadsheet,

Appendix_7_Posterior_WTP_ratio_method.xlsx.

Online Appendix 3.8.
Additional Citations: Five Marketing Science Papers that Discuss Scale Explicitly

Fiebig DG, Keane MP, Louviere J, Wasi N (2010) The generalized multinomial logit model: Accounting for scale and coefficient heterogeneity. *Marketing Science* 29(3):393-421.

Gilbride TJ, Lenk PJ, Brazell JD (2008) Market share constraints and the loss function in choice-based conjoint analysis. *Marketing Science* 27(6):995-1011.

Narayan V, Rao VR, Saunders C (2011) How peer influence affects attribute preferences: A Bayesian Updating Mechanism, *Marketing Science* 30(2):368-384.

Salisbury LC, Feinberg FM (2010) Alleviating the constant stochastic variance assumption in decision research: Theory, measurement, and experimental test. *Marketing Science* 29(1):1-17.

Swait J, Erdem T (2007) Brand effect on choice and choice set formation under uncertainty. *Marketing Science* 26(5): 679-697.